Quantum structure in cognition: Why and how concepts are entangled

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Abstract

One of us has recently elaborated a theory for modelling concepts that uses the state context property (SCoP) formalism, i.e. a generalization of the quantum formalism. This formalism incorporates context into the mathematical structure used to represent a concept, and thereby models how context influences the typicality of a single exemplar and the applicability of a single property of a concept. The notion of 'state of a concept' is introduced to account for this contextual influence, which proposes a solution for the *Pet-Fish problem* and several difficulties occurring in concept theory. Then, a quantum model has been worked out which reproduces the membership weights of several exemplars of concepts and their combinations. We show in this paper that a further relevant effect appears in a natural way whenever two or more concepts combine, namely, *entanglement*. The presence of entanglement is explicitly revealed by considering a specific example with two concepts, constructing some Bell's inequalities for this example, testing them in a real experiment with test subjects, and finally proving that Bell's inequalities are violated in this case. We show that the intrinsic and unavoidable character of entanglement can be explained in terms of the weights of the exemplars of the combined concept with respect to the weights of the exemplars of the component concepts, and elaborate a concrete quantum model for the proposed example.

Keywords: Concept combination, Bell's inequalities, entanglement, quantum cognition

1 Introduction

Understanding the mechanism of how concepts combine to form sentences and texts so that it is possible to communicate meaning among human minds is one of the major challenges in the psychological studies on human thought. None of the existing theories on concepts explains however 'how concepts combine'. This *combination problem* was manifestly revealed by Hampton's experiments [1, 2] which measured the deviation from classical set theoretic membership weights of exemplars with respect to pairs of concepts and their conjunction or disjunction. Hampton's investigation was motivated by the so-called *Guppy effect* in concept conjunction found by Osherson and Smith [3]. These authors considered the concepts *Pet* and *Fish* and their conjunction *Pet-Fish*, and observed that, while an exemplar such as *Guppy* was a very typical example of *Pet-Fish*, it was neither a very typical example of *Pet* nor of *Fish*. Hence, the typicality of a specific exemplar with respect to the conjunction of concepts shows an unexpected behavior from the point of view of classical set and probability theory. As a result of this work, the problem is often referred to as the *Pet-Fish problem* and the effect is usually called the *Guppy effect*. Hampton identified a Guppy-like

effect for the membership weights of exemplars with respect to pairs of concepts and their conjunction [1], and equally so for the membership weights of exemplars with respect to pairs of concepts and their disjunction [2]. Several experiments have since been conducted (see, e.g., [4]) and many approaches have been propounded to solve the Pet-Fish problem (see, e.g., fuzzy set based theories [5, 6, 7]) and to provide a satisfactory mathematical model of concept combinations (see, e.g., explanation based theories [8, 9, 10]). But none of the currently existing concepts theories provide a satisfactory description or explanation of such effects [4, 9, 10].

Inspired by a formalism providing an operational foundation of quantum mechanics [11, 12, 13, 14], one of the authors has elaborated, together with some co-workers, a *State Context Property* (*SCoP*) formalism to model and represent concepts [15, 16, 17, 18]. In the SCoP formalism each concept is associated with well defined sets of states, contexts and properties. Concepts change continuously under the influence of a context and this change is described by a change of the state of the concept. For each exemplar of a concept, the typicality varies with respect to the context that influences it. Analogously, for each property, the applicability varies with respect to the context. This implies the presence of both a *contextual typicality* and an *applicability effect*. The *Pet-Fish problem* is solved in the SCoP formalism because in different combinations the concepts are in different states. In particular, in the combination *Pet-Fish* the concept *Pet* is in a state under the context *The Pet is a Fish*. The state of *Pet* under the context *The Pet is a Fish* has different typicalities, which explains the guppy effect. On the basis of the SCoP formalism, a mathematical model using the formalism of quantum mechanics in Hilbert space has been worked out which allows one to reproduce the experimental results obtained by Hampton on conjunctions and disjunctions of concepts. This formulation identifies the presence of typically quantum effects in the mechanism of combination of concepts, e.g., contextual influence, superposition, interference, emergence, etc. [19, 20, 21, 22, 23, 24, 25].

In this paper we show that another relevant effect which is usually considered as characteristic of quantum mechanical entities, that is, *entanglement*, is present whenever two or more concepts combine to form sentences. The presence of entanglement is explicitly revealed by considering two concepts, i.e. *Animal* and *Acts*, and their combination *The Animal Acts*, together with some exemplars *Horse*, *Bear*, *Tiger*, *Cat* (for *Animal*) and *Growls*, *Whinnies*, *Snorts*, *Meows* (for *Acts*), and constructing some Bell's inequalities in the version derived by Clauser, Horne, Shimony and Holt [26] (Sec. 2). We then test these Bell's inequalities in a real experiment with 81 test subjects and analyze the obtained data (Sec. 3). The experiment shows a significant violation of Bell's inequalities, hence it proves the entanglement between the concepts *Animal* and *Acts* when they form the sentence *The Animal Acts*. Moreover, we compare the obtained data with the results that would have been obtained if context and meaning had not influenced the subjects' minds. In the latter case, indeed, Bell's inequalities are not violated, hence their violation in our experiment shows that meaning and context play a basic role in the combination of concepts. We finally provide an explanation of the origins and the ubiquity of entanglement in combined concepts in terms of weights of the exemplars of the combined concept with respect to the weights of the exemplars of the component concepts. This explanation is supported by an explicit quantum model for the proposed experiment (Sec. 4).

We conclude this section by observing that the potentially fundamental role played by entanglement in word association was pointed out by Nelson and McEvoy and Bruza et al. in [27, 28]. In [29] it is shown that if one assumes that words can become entangled in the human mental lexicon, then one can provide a unified framework in which two seemingly competing approaches for modeling the activation level of words in human memory, namely, the Spreading Activation and the Spooky-activation-at-a-distance, can be recovered.

2 Detecting entanglement between concepts

We illustrate in this section how entanglement appears in a natural way whenever two or more concepts combine. To this aim, we analyze here an example with two concepts and a combination along the lines put forward in [16, 17, 18].

We regard the sentence *The Animal Acts* as a conceptual entity, hence as a combination of the concepts *Animal* and *Acts*. Then, we show the presence of entanglement between these two concepts by testing Bell's inequality with respect to them. We consider two couples of exemplars or states of the concept *Animal*, namely *Horse*, *Bear* and *Tiger*, *Cat*, and also two couples of exemplars or states of the concept *Acts*, namely *Growls*, *Whinnies* and *Snorts*, *Meows* – for our experiment we specifically consider forms of actions, hence exemplars of *Acts*, which consists of possible animal sounds, hence exemplars of *Making A Sound*. Our first experiment A consists in test subjects choosing between the two exemplars *Horse* and *Bear* to answer the question 'is a good example of' the concept *Animal*, and we put $E(A) = +1$ if *Horse* is chosen, hence the state of *Animal* changes to *Horse*, and $E(A) = -1$ if *Bear* is chosen, hence the state of *Animal* changes to *Bear*, introducing in this way the function E which measures the 'expectation value' for the test outcomes concerned. Our second experiment A' consists in test subjects choosing between the two exemplars *Tiger* and *Cat* to answer the question 'is a good example of' the concept *Animal*, and we consistently put $E(A') = +1$ if *Tiger* is chosen and $E(A') = -1$ if *Cat* is chosen to introduce a measure of the expectation value. The third experiment B consists in test subjects choosing between the two exemplars *Growls* and *Whinnies* to answer the question 'is a good example of' the concept *Acts*, with $E(B) = +1$ if *Growls* is chosen and $E(B) = -1$ if *Whinnies* is chosen. The fourth experiment B' consists in test subjects choosing between the exemplars *Snorts* and *Meows* to answer the question 'is a good example of' the concept *Acts*, with $E(B') = +1$ if *Snorts* is chosen and $E(B') = -1$ if *Meows* is chosen.

Let us now consider coincidence experiments in combinations AB, $A'B$, AB' and $A'B'$ for the conceptual combination *The Animal Acts*. Concretely, this means that, for example, test subjects taking part in the experiment AB, to answer the question 'is a good example of', will choose between the four possibilities (1) *The Horse Growls*, (2) *The Bear Whinnies* – and if one of these is chosen we put $E(AB) = +1$ – and (3) *The Horse Whinnies*, (4) *The Bear Growls* – and if one of these is chosen we put $E(AB) = -1$. For the coincidence experiment, $A'B$ subjects, to answer the question 'is a good example of', will choose between (1) *The Tiger Growls*, (2) *The Cat Whinnies* – and in case one of these is chosen we put $E(A'B) = +1$ – and (3) *The Tiger Whinnies*, (4) *The Cat Growls* – and in case one of these is chosen we put $E(A'B) = -1$. For the coincidence experiment, AB' subjects, to answer the question 'is a good example of', choose between (1) *The Horse Snorts*, (2) *The Bear Meows* – and in case one of these is chosen we put $E(AB') = +1$ – and (3) *The Horse Meows*, (4) *The Bear Snorts* – and in case one of these is chosen we put $E(AB') = -1$. And finally, for the coincidence experiment, $A'B'$ subjects, to answer the question 'is a good example of', will choose between (1) *The Tiger Snorts*, (2) *The Cat Meows* – and in case one of these is chosen we put $E(A'B') = +1$ – and (3) *The Tiger Meows*, (4) *The Cat Snorts* – and in case one of these is chosen we put $E(A'B') = -1$. We can now evaluate the expectation values $E(A', B')$, $E(A', B)$, $E(A, B')$ and $E(A, B)$ associated with the coincidence experiments $A'B'$, $A'B$, AB' and AB, respectively, and substitute them into the Clauser-Horne-Shimony-Holt variant of Bell's inequality [26]

$$-2 \leq E(A', B') + E(A', B) + E(A, B') - E(A, B) \leq 2. \tag{1}$$

From the well-known analysis of Bell's inequality follows that in case the experimental expectation values violate Eq. (1), a local and classical probabilistic description is not possible, and entanglement exists between the given concepts.

We note that the maximum violation of the Bell's inequality in Eq. (1) occurs when the quantity $E(A', B') + E(A', B) + E(A, B') - E(A, B)$ is equal to $+4$, that is, when the outcome for each one of the

members of this expression is +1, +1, +1 and -1. Let us make an intuitive analysis of the situation such that we can see why Bell's inequality will most probably be violated for our experiment. In the coincidence experiment AB, both *The Horse Whinnies* and *The Bear Growls* will yield rather high scores, with the two remaining possibilities *The Horse Growls* and *The Bear Whinnies* being chosen little. This means that we will get $E(A, B)$ close to -1. On the other hand, in the coincidence experiment $A'B$ one of the four choices will be prominent, namely *The Tiger Growls*, while the three other possibilities, *The Cat Whinnies*, *The Tiger Whinnies*, and *The Cat Growls*, will be much less present amongst the choices made by the test subjects. This means that we have $E(A', B)$ close to +1. In the two remaining coincidence experiments, we equally have that only one of the choices is prominent. For A, B', this is *The Horse Snorts*, with the other three *The Bear Meows*, *The Horse Meows* and *The Bear Snorts* being much less present. For $A'B'$, the prominent choice is *The Cat Meows*, while the other three *The Tiger Snorts*, *The Tiger Meows* and *The Cat Snorts* are much less present. This means that we have $E(A, B')$ is close to +1 and $E(A', B')$ is close to +1. Coming to the expectation values, we hence can expect that Eq. (1) be violated, and that case (ii) occurs such that the existence of entanglement between the considered concepts would be proven.

One of us has recently shown [30] that Eq. (1) is violated in the concept combination *The Animal Acts* by using the World Wide Web as a conceptual domain. In the next section we will show that a violation occurs also when the data are collected from a real experiment with test subjects following the standard procedure of psychology experiments in concept research.

3 Description of the experiment

The entanglement mentioned in the foregoing section was tested in an experiment where 81 participating subjects were presented with a questionnaire to be filled out accompanied by the following text:

This study has to do with what we have in mind when we use words that refer to categories, and more specifically 'how we think about examples of categories'. Let us illustrate what we mean. Consider the category 'fruit'. Then 'orange' and 'strawberry' are two examples of this category, but also 'fig' and 'olive' are examples of the same category. In each test of the questionnaire you will be asked to pick one of the examples of a set of given examples for a specific category. And we would like you to pick that example that you find 'a good example' of the category. In case there are more than one example which you find a good example, pick then the one you find the best of all the good examples. In case there are two examples which you both find equally good, and hence hesitate which ones to take, just take then the one you slightly prefer, however slight the preference might be. It is mandatory that you always 'pick one and only one example', hence in case of doubt, anyhow pick one and only one example. This is necessary for the experiment to succeed. So, one of the tests could be that the category 'fruit' is given, and you are asked to pick one of the examples 'orange', 'strawberry', 'fig' or 'olive' as a good example, and in case of doubt the best of the ones you doubt about, and in case you cannot decide, pick one anyhow. Let all aspects of yourself play a role in the choice you make, ratio, but also imagination, feeling, emotion, and whatever.

Let us now examine the obtained results.

For the coincidence experiment AB, 4 subjects chose the example *The Horse Growls* as a good example of the combination *The Animal Acts*, 5 subjects chose *The Bear Whinnies*, 51 subjects chose *The Horse Whinnies*, and 21 subjects chose *The Bear Growls*. This means that on a totality of 81 test subjects we get fractions of 4, 5, 51 and 21 for the different combinations considered. This allows us to calculate the probability for one of the combinations to be chosen. We have $P(A_1, B_1) = 4/81 = 0.0494$ for *The Horse Growls*, $P(A_2, B_2) = 21/81 = 0.2593$ for *The Bear Whinnies*, $P(A_1, B_2) = 51/81 = 0.6296$ for *The Horse Whinnies* and $P(A_2, B_1) = 5/81 = 0.0617$ for *The Bear Growls*. Knowing these probabilities, we can again calculate the expectation value for this coincidence experiment by means of the equation

$E(A,B) = P(A_1,B_1) + P(A_2,B_2) - P(A_2,B_1) - P(A_1,B_2) = -0.7778$. We calculate the expectation values $E(A',B)$, $E(A,B')$ and $E(A',B')$ in an analogous way. For the coincidence experiment $A'B$, 63 subjects chose the example *The Tiger Growls* as a good example of the combination *The Animal Acts*, 4 subjects chose *The Cat Whinnies*, 7 subjects chose *The Tiger Whinnies*, and 7 subjects chose *The Cat Growls*. This gives $P(A'_1,B_1) = 0.7778$, $P(A'_2,B_2) = 0.0494$, $P(A'_1,B_2) = 0.0864$ and $P(A'_2,B_1) = 0.0864$, hence $E(A',B) = 0.6543$. For the coincidence experiment AB', 48 subjects chose the example *The Horse Snorts* as a good example of the combination *The Animal Acts*, 7 subjects chose *The Bear Meows*, 2 subjects chose *The Horse Meows*, and 24 subjects chose *The Bear Snorts*. This gives $P(A_1,B'_1) = 0.5926$, $P(A_2,B'_2) = 0.0864$, $P(A_1,B'_2) = 0.0247$ and $P(A_2,B'_1) = 0.2963$, hence $E(A,B') = 0.3580$. For the coincidence experiment $A'B'$, 12 subjects chose the example *The Tiger Snorts* as a good example of the combination *The Animal Acts*, 54 subjects chose *The Cat Meows*, 7 subjects chose *The Tiger Meows*, and 8 subjects chose *The Cat Snorts*. This gives $P(A'_1,B'_1) = 0.1481$, $P(A'_2,B'_2) = 0.6667$, $P(A'_1,B'_2) = 0.0864$ and $P(A'_2,B'_1) = 0.0988$, hence $E(A',B') = 0.6296$. For the expression appearing in the Clauser-Horne-Shimony-Holt variant of Bell's inequalities, we get

$$E(A',B') + E(A',B) + E(A,B') - E(A,B) = 2.4197 \tag{2}$$

which is manifestly greater than 2, hence it violates Bell's inequalities and reveals entanglement between the concept *Animal* and the concept *Acts*.

The above violation of Bell's inequalities constitutes our main result in this paper and we will exhaustively comment on it in the next section. But we first want to consider Bell's inequalities under different perspectives.

Suppose that there are two separated sources of knowledge, e.g., two test subjects, and consider the coincidence experiment AB. Let $P(A_1)$ be the probability that the first subject choose the exemplar *Horse* as a good example of the concept *Animal*, let $P(B_1)$ be the probability that the second subject choose the exemplar *Growls* as a good example of the concept *Acts*, and let us estimate the probability that the example *The Horse Growls* be a good example of the conceptual combination *The Animal Acts* as the product $P(A_1)P(B_1)$, that is, as the joint probability $P_{prod}(A_1,B_1)$ that the first subject choose *Horse* and the second subject choose *Growls*. By referring to the experimental data that have been collected we have $P(A_1) = 43/81 = 0.5309$, $P(B_1) = 39/81 = 0.4815$, $P_{prod}(A_1,B_1) = P(A_1)P(B_1) = 0.2556$. Analogously, we can calculate the probability that *The Bear Whinnies* be a good example of *The Animal Acts* as the product of the probability $P(A_2)$ that the first subject choose *Bear* as a good example of *Animal* times the probability $P(B_2)$ that the second subject choose *Whinnies* as a good example of *Acts*. We find from empirical data $P(A_2) = 38/81 = 0.4691$, $P(B_2) = 42/81 = 0.5185$, hence $P_{prod}(A_2,B_2) = P(A_2)P(B_2) = 0.2433$. By proceeding in an analogous way we calculate the probability that *The Horse Whinnies* be a good example of *The Animal Acts* as the product of the probability $P(A_1)$ that the first subject choose *Horse* as a good example of *Animal* times the probability $P(B_2)$ that the second subject choose *Whinnies* as a good example of *Acts*. We find $P_{prod}(A_1,B_2) = P(A_1)P(B_2) = 0.2753$. Furthermore, if we calculate the probability that *The Bear Growls* be a good example of *The Animal Acts* as the product of the probability $P(A_2)$ that the first subject choose *Bear* as a good example of *Animal* times the probability $P(B_1)$ that the second subject choose *Growls* as a good example of *Acts*, we find $P_{prod}(A_2,B_2) = P(A_2)P(B_1) = 0.2259$. The expectation value is $E_{prod}(A,B) = P_{prod}(A_1,B_1) + P_{prod}(A_2,B_2) - P_{prod}(A_2,B_1) - P_{prod}(A_1,B_2) = -0.0022$. Let us now consider the coincidence experiment $A'B$. The probability that the first subject choose the example *Tiger* as a good example of *Animal* is $P(A'_1) = 59/81 = 0.7284$, while the probability that the first subject choose *Cat* as a good example of *Animal* is $P(A'_2) = 22/81 = 0.2716$. If we calculate the probability that *The Tiger Growls* be a good example of *The Animal Acts* as the product of the probability $P(A'_1)$ that the first subject choose *Tiger* as a good example of *Animal* times the probability $P(B_1)$ that the second subject choose *Growls* as a good example of *Acts*, we find $P_{prod}(A'_1,B_1) = P(A'_1)P(B_1) =$

0.3507. Analogously, we find $P_{prod}(A'_2, B_2) = P(A'_2)P(B_2) = 0.1408$, $P_{prod}(A'_1, B_2) = P(A'_1)P(B_2) = 0.3777$, $P_{prod}(A'_2, B_1) = P(A'_2)P(B_1) = 0.1308$. The expectation value is $E_{prod}(A', B) = P_{prod}(A'_1, B_1) + P_{prod}(A'_2, B_2) - P_{prod}(A'_2, B_1) - P_{prod}(A'_1, B_2) = -0.0169$. Let us come to the coincidence experiment AB'. The probability that the second subject choose the example *Snorts* as a good example of *Acts* is $P(B'_1) = 26/81 = 0.3210$, while the probability that the second subject choose *Meows* as a good example of *Acts* is $P(B'_2) = 55/81 = 0.6790$. If we calculate the probability that *The Horse Snorts* be a good example of *The Animal Acts* as the product of the probability $P(A_1)$ that the first subject choose *Horse* as a good example of *Animal* times the probability $P(B'_1)$ that the second subject choose *Snorts* as a good example of *Acts*, we find $P_{prod}(A_1, B'_1) = P(A_1)P(B'_1) = 0.1704$. Analogously, we find $P_{prod}(A_1, B'_1) = P(A_2)P(B'_2) = 0.3185$, $P_{prod}(A_1, B'_1) = P(A_1)P(B'_2) = 0.3605$, $P_{prod}(A_1, B'_1) = P(A_2)P(B'_1) = 0.1506$. The expectation value is $E_{prod}(A, B') = P_{prod}(A_1, B'_1) + P_{prod}(A_2, B'_2) - P_{prod}(A_2, B'_1) - P_{prod}(A_1, B'_2) = -0.0221$. Finally, let us consider the coincidence experiment $A'B'$. If we calculate the probability that *The Tiger Snorts* be a good example of *The Animal Acts* as the product of the probability $P(A'_1)$ that the first subject choose *Tiger* as a good example of *Animal* times the probability $P(B'_1)$ that the second subject choose *Snorts* as a good example of *Acts*, we find $P_{prod}(A'_1, B'_1) = P(A'_1)P(B'_1) = 0.2338$. Analogously, we find $P_{prod}(A'_2, B'_2) = P(A'_2)P(B'_2) = 0.1844$, $P_{prod}(A'_1, B'_2) = P(A'_1)P(B'_2) = 0.4946$, $P_{prod}(A'_2, B'_1) = P(A'_2)P(B'_1) = 0.0871$. The expectation value is $E_{prod}(A', B') = P_{prod}(A'_1, B'_1) + P_{prod}(A'_2, B'_2) - P_{prod}(A'_2, B'_1) - P_{prod}(A'_1, B'_2) = -0.1635$. The 'product' expectation values $E_{prod}(A, B)$, $E_{prod}(A', B)$, $E_{prod}(A, B')$ and $E_{prod}(A', B')$ can then be put into the Bell inequality, which gives

$$E_{prod}(A', B') + E_{prod}(A', B) + E_{prod}(A, B') - E_{prod}(A, B) = -0.2003. \tag{3}$$

This result is very different from the earlier obtained expression, and also does not violate Bell's inequalities. The reason for this is that in the case of 'separated sources of knowledge', the non-violation of Bell's inequalities is structural [30]. The foregoing considerations show that the crucial matter in the violation of Bell's inequalities is the non-product nature of the probabilities $P(A_i, B_j)$, $P(A'_i, B_j)$, $P(A_i, B'_j)$ and $P(A'_i, B'_j)$, e.g., $P(A_i, B_j) \neq P(A_i)P(B_j)$.

Indeed, consider for example the probability $P(A_1, B_1)$ and let us analyze why it is different from $P(A_1)P(B_1)$. We have that $P(A_1, B_1)$ is the probability, empirically estimated, that a given test subject choose the sentence *The Horse Growls* as a good example of the concept *The Animal Acts*, and then we find $P(A_1, B_1) = 0.0494$. On the contrary, $P(A_1)P(B_1)$ is the probability that, of two given test subjects, the first choose *Horse* as a good example of *Animal* and the other choose independently *Growls* as a good example of *Acts*, and then we find $P(A_1)P(B_1) = 0.2556$. These values are very different and it is easy to understand why. The probability to find the sentence part *The Horse Growls* is little, because any meaning this sentence may have will not be easily ascertained, since it is most unusual for horses to growl. If however two 'separated' or 'independent' subjects are chosen at random, the probability that *Horse* be chosen by one of them, and *Growls* be chosen by the other, is substantial. The fundamental reason for this difference is that in the second case the choices are 'separated' or 'independent' or, rather, 'not connected by meaning'.

The results above show that 'meaning' plays a fundamental role in determining the experimental weights of the examples of concept combinations. But, there are stronger arguments to maintain that context and meaning are crucial in human thought, hence a combination of concepts is not like a 'bag of words', as implied by the mathematical structure of existing semantic theories, e.g., LSA.

Let us calculate the data that would have been obtained if the minds of the test subjects had not been influenced by context and meaning. Consider the coincidence experiment AB and suppose that a given subject chooses the exemplar *Horse* as a good example of the concept *Animal* and *Growls* as a good example of the concept *Acts*. Should context and meaning not play any role, then the subject would choose with certainty the example *The Horse Growls* as a good example of the combination *The Animal Acts*.

We can thus evaluate the probability $P_{class}(A_1, B_1)$ that a given subject choose *Horse* in the experiment A and *Growls* in the experiment B. It is given by $P_{class}(A_1, B_1) = 19/81 = 0.2346$, where 19 is the number of subjects who chose *Horse* in the experiment A and *Growls* in the experiment B. This probability can be used as an estimation of the probability that a given subject choose *The Horse Growls* as a good example of *The Animal Acts*. We can repeat the same reasoning for the other possible results in the coincidence experiment AB, thus getting $P_{class}(A_2, B_2) = 0.2222$, $P_{class}(A_1, B_2) = 0.2963$ and $P_{class}(A_2, B_1) = 0.2469$. Hence the expectation value is $E_{class}(A, B) = P_{class}(A_1, B_1) + P_{class}(A_2, B_2) - P_{class}(A_1, B_2) - P_{class}(A_2, B_1) = -0.0864$ in this case. Analogously, we get $E_{class}(A', B) = 0.1235$, $E_{class}(A, B') = -0.0123$ and $E_{class}(A', B') = -0.1111$ for the expectation values of the other coincidence experiments. The 'classical' expectation values $E_{class}(A, B)$, $E_{class}(A', B)$, $E_{class}(A, B')$ and $E_{class}(A', B')$ can then be inserted into the Bell inequality, which gives

$$E_{class}(A', B') + E_{class}(A', B) + E_{class}(A, B') - E_{class}(A, B) = 0.0864. \tag{4}$$

As we can see, the obtained value does not violate Bell's inequalities. As a consequence, the violation of Bell's inequalities in the experiment that we have considered can be interpreted as proving that meaning and context are fundamental for the mechanism of construction of sentences.

To conclude this section we observe that we also performed a statistical analysis of the empirical data using the 't-test for paired two samples for means' to estimate the probability that the shifts from Bell's inequalities be due to chance. The p-values that have been calculated are very small in all cases, which allow one to state that the deviation effects are not caused by random fluctuations.

4 Explanation of entanglement and a Hilbert space model

A fundamental consequence of the experimental results obtained in Sec. 3 is that any formalism aiming at representing concepts should incorporate the possibility of having entangled concepts from the very beginning. In order to explain the mechanism of entanglement between concepts together with the deep causes of its ubiquity let us represent concepts by using the SCoP formalism [16, 17]. Let us consider the conceptual entity *Animal*. This is an abstract entity obtained from all possible concrete examples of animals, e.g., *Horse*, *Bear*, *Tiger*, *Cat*, etc. When we ask a subject to estimate whether a given example, say, *Horse* is a 'good example' of the concept *Animal* this operation corresponds to performing a measurement: in the SCoP formalism this measurement is described by means of a 'collapse' of the entity *Animal* from its ground state p_{Animal} to the pure state p_{Horse}. This collapse procedure is a transition from abstract to more concrete. The concept *Animal* is then connected with the concrete examples of animals by weights, that is, the frequencies, empirically estimated, which each concrete animal appear. Analogously, the concept *Acts* is connected with the concrete actions of animals by empirical weights. Let us now consider the combination *The Animal Acts*. It is connected with the concrete examples *The Horse Growls*, *The Tiger Meows*, etc., by weights too. But, the weight of, say, *The Horse Growls* is not the product of the weight of *Horse* in *Animal* times the weight of *Growls* in *Acts* in this case, otherwise Bell's inequalities would have been satisfied, and this explains the presence of entanglement between *Animal* and *Acts*.

It is important to note that the origin of entanglement is due to the fact that the combination *The Animal Acts* is a new concept or, equivalently, a new entity, and the reality of the entity is represented by the pure states, rather than by the weights, which are instead related to context, hence to the interaction of the entity with the external world. As a consequence, entanglement in concepts does not strictly depend on the linearity of the tensor product Hilbert space that can be used to model the entity *The Animal Acts* – we remind that the *Tsirelson inequalities* [31] hold in the specific case that we have considered, therefore a purely quantum model can be worked out in this case. However, the presence of a quantum model can be useful to represent mathematically how concepts entangle. More concretely, let us denote the ground

states of the concepts *Animal* and *Acts* by the unit vectors $|p_{Animal}\rangle$ and $|p_{Acts}\rangle$, respectively. Since *Animal* and *Acts* are both abstractions of, say, *Horse* and *Bear* and of *Growls* and *Whinnies*, respectively, we have

$$|p_{Animal}\rangle = a_1|p_H\rangle + a_2|p_B\rangle, \quad |p_{Acts}\rangle = b_1|p_G\rangle + b_2|p_W\rangle \tag{5}$$

where $|a_1|^2$ and $|a_2|^2$, and $|b_1|^2$ and $|b_2|^2$, respectively, are the weights that both concretizations carry, and the unit vectors $|p_H\rangle$, $|p_B\rangle$, $|p_G\rangle$ and $|p_W\rangle$ represent the states p_{Horse}, p_{Bear}, p_{Growls} and $p_{Whinnies}$, respectively. The ground state $p_{The\ Animal\ Acts}$ of the combination *The Animal Acts*, being an abstraction of 'all combinations of the concrete cases', is then represented by the unit vector

$$|p_{The\ Animal\ Acts}\rangle = c_1|p_{HG}\rangle + c_2|p_{BW}\rangle + c_3|p_{HW}\rangle + c_4|p_{BG}\rangle, \tag{6}$$

where the unit vectors $|p_{HG}\rangle$, $|p_{BW}\rangle$, $|p_{HW}\rangle$ and $|p_{BG}\rangle$ represent the states $p_{The\ Horse\ Growls}$, $p_{The\ Bear\ Whinnies}$, $p_{The\ Horse\ Whinnies}$ and $p_{The\ Bear\ Growls}$, respectively. Eq. (6) is not, in general, a product, hence it is not equal to the tensor product $|p_{Animal}\rangle \otimes |p_{Acts}\rangle$, which is the mathematical basis of the presence of entanglement.

The above explanation can be better understood by constructing a concrete quantum model for the concept combination *The Animal Acts* in the tensor product Hilbert space $\mathbb{C}^2 \otimes \mathbb{C}^2$, where $\mathbb{C}$ is the field of complex numbers. Let the concepts *Animal* and *Acts* be both associated with the Hilbert space $\mathbb{C}^2$ and the combination *The Animal Acts* be associated with the tensor product Hilbert space $\mathbb{C}^2 \otimes \mathbb{C}^2$. Let $\{|p_H\rangle, |p_B\rangle\}$ and $\{|p_G\rangle, |p_W\rangle\}$ be orthonormal (ON) bases in $\mathbb{C}^2$, and let us put $|p_H\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|p_B\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $|p_G\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|p_W\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Eq. (6) can then be rewritten as

$$|p_{The\ Animal\ Acts}\rangle = c_1|p_H\rangle|p_G\rangle + c_2|p_B\rangle|p_W\rangle + c_3|p_H\rangle|p_W\rangle + c_4|p_B\rangle|p_G\rangle, \tag{7}$$

where $c_1 = \sqrt{P(A_1, B_1)}e^{i\theta_1}$, $c_2 = \sqrt{P(A_2, B_2)}e^{i\theta_2}$, $c_3 = \sqrt{P(A_1, B_2)}e^{i\theta_3}$, $c_4 = \sqrt{P(A_2, B_1)}e^{i\theta_4}$ (see Sec. 3), and $\theta_i \in \Re$. Let us now assume that also $\{|p_T\rangle, |p_C\rangle\}$ and $\{|p_S\rangle, |p_M\rangle\}$ are ON bases in $\mathbb{C}^2$. Then, $|p_{The\ Animal\ Acts}\rangle$ can alternatively be decomposed as follows

$$|p_{The\ Animal\ Acts}\rangle = d_1|p_T\rangle|p_G\rangle + d_2|p_C\rangle|p_W\rangle + d_3|p_T\rangle|p_W\rangle + d_4|p_C\rangle|p_G\rangle \tag{8}$$
$$|p_{The\ Animal\ Acts}\rangle = e_1|p_H\rangle|p_S\rangle + e_2|p_B\rangle|p_M\rangle + e_3|p_H\rangle|p_M\rangle + e_4|p_B\rangle|p_S\rangle \tag{9}$$
$$|p_{The\ Animal\ Acts}\rangle = f_1|p_T\rangle|p_S\rangle + f_2|p_C\rangle|p_M\rangle + f_3|p_T\rangle|p_M\rangle + f_4|p_C\rangle|p_S\rangle \tag{10}$$

where $d_1 = \sqrt{P(A_1', B_1)}e^{i\phi_1}$, $d_2 = \sqrt{P(A_2', B_2)}e^{i\phi_2}$, $d_3 = \sqrt{P(A_1', B_2)}e^{i\phi_3}$, $d_4 = \sqrt{P(A_2', B_1)}e^{i\phi_4}$, $e_1 = \sqrt{P(A_1, B_1')}e^{i\alpha_1}$, $e_2 = \sqrt{P(A_2, B_2')}e^{i\alpha_2}$, $e_3 = \sqrt{P(A_1, B_2')}e^{i\alpha_3}$, $e_4 = \sqrt{P(A_2, B_1')}e^{i\alpha_4}$, $f_1 = \sqrt{P(A_1', B_1')}e^{i\beta_1}$, $f_2 = \sqrt{P(A_2', B_2')}e^{i\beta_2}$, $f_3 = \sqrt{P(A_1', B_2')}e^{i\beta_3}$, $f_4 = \sqrt{P(A_2', B_1')}e^{i\beta_4}$ (see Sec. 3), and $\phi_i, \alpha_i, \beta_i \in \Re$. Moreover, we can write, with $\theta, \phi, \alpha, \beta \in \Re$

$$|p_T\rangle = \cos\tfrac{\theta}{2}e^{-i\frac{\phi}{2}}|p_H\rangle + \sin\tfrac{\theta}{2}e^{i\frac{\phi}{2}}|p_B\rangle, |p_C\rangle = -\sin\tfrac{\theta}{2}e^{-i\frac{\phi}{2}}|p_H\rangle + \cos\tfrac{\theta}{2}e^{i\frac{\phi}{2}}|p_B\rangle \tag{11}$$
$$|p_S\rangle = \cos\tfrac{\alpha}{2}e^{-i\frac{\beta}{2}}|p_G\rangle + \sin\tfrac{\alpha}{2}e^{i\frac{\beta}{2}}|p_W\rangle, |p_M\rangle = -\sin\tfrac{\alpha}{2}e^{-i\frac{\beta}{2}}|p_G\rangle + \cos\tfrac{\alpha}{2}e^{i\frac{\beta}{2}}|p_W\rangle \tag{12}$$

If we now insert Eqs. (11)–(12) into Eqs. (8)–(10) and equalize Eq. (7) with Eqs. (8), (9) and (10), respectively, we find a system of 12 algebraic equations in the 20 real variables $\theta_i, \phi_i, \alpha_i, \beta_i, \theta, \phi, \alpha, \beta$. The system, once solved, provides the required quantum model in Hilbert space.

The unavoidability of entanglement could explain the difficulties that scholars encounter in putting forward a modeling scheme for concepts and their combinations in which individual concepts are represented

by a unique mathematical structure, e.g., vectors such as in LSA, without introducing the tensor product structure (see, e.g., [32]).

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